Discrete Mathematics 54 (1985) 125–126
North-Holland

RESEARCH PROBLEMS

With Volume 36 of Discrete Mathematics, a Research Problem Section has been established. Problems in this section are intended to be research level problems rather than standard exercises. People wishing to submit such problems should send them (in duplicate) to:

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The following should be included: (1) The name of the person(s) who originally posed the problem; (2) the name and address of a person willing to act as a correspondent; and (3) references and other pertinent information.

The Editorial Board of Discrete Mathematics invites readers to provide information about solutions, partial results and other pertinent items related to problems posed earlier, if possible indicating the source of the information, for example papers appearing in different journals, preprints, etc. This information will be passed along to readers from time to time in order to keep them appraised of the current status of various problems.

People wishing to provide information about problems that appeared earlier should write to Professor Alspach. People wishing to correspond on technical matters concerning a problem should write to the correspondent.

Problem 63. Posed by Martin Grötschel and W.R. Pulleyblank.

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If $G=(V,E)$ is a graph, let R^E denote euclidean $|E|$-space with coordinates indexed by E. If $F\subseteq E$ is a subset of edges of G, then the vector $x^F\in R^E$ satisfying $x_e^F=1$ if $e\in F$ and $x_e^F=0$ if $e\notin F$ is called the *incidence vector* of F. The polytope

$$P_B(G) = \text{conv}\{x^F \in R^E\colon\ (V, F) \text{ is a bipartite subgraph of } G\}$$

is called the *bipartite subgraph polytope* of G.

Consider the following two systems of inequalities:

$$0 \leqslant x_e \leqslant 1, \quad e \in E, \tag{1}$$

and

$$\sum_{e \in C} x_e = |C| - 1, \quad C \text{ an odd cycle in } G. \tag{2}$$

A graph $G = (V, E)$ which satisfies

$$P_B(G) = \{x \in R^E\colon\ x \text{ satisfies (1) and (2)}\}$$

is called *weakly bipartite*. A graph which is not weakly bipartite is called *strongly nonbipartite*. If G is strongly nonbipartite and every subgraph of G obtained by removing one edge is weakly bipartite, then G is called *minimally strongly nonbipartite*. For example, the complete graph K_5 and all graphs obtained from K_5 by replacing one (or more) edges by a path of odd length are minimally strongly nonbipartite.

Give further characterizations of minimally strongly nonbipartite graphs.

Problem 64. Posed by Martin Grötschel.

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It is trivial to decide whether or not a graph contains an odd cycle. Is there a polynomial time algorithm to decide whether a graph contains an odd hole (that is, an odd cycle of length at least 5 without chords)? This problem can be solved in polynomial time for planar graphs (see [1]).

Reference

[1] Wen-Lian Hsu, On the existence of induced odd cycles in planar graphs, Department of Industrial Engineering and Management Sciences, Northwestern University, Evanston, IL, USA (1984).